LEARNING MATERIAL
ON
TRIGONOMETRY

SEMESTER : I
DEPARTMENT : MATHEMATICS AND SCIENCE
SUBJECT NAME : ENGINEERING MATHEMATICS-I
SUBJECT CODE : TH.3

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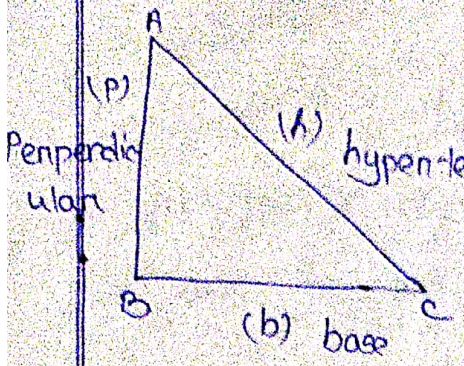
Trigonmetry

(Measurement of angles)



Right angle triangle

$$\boxed{h^2 = p^2 + b^2}$$

↓

Pythagoras theorm

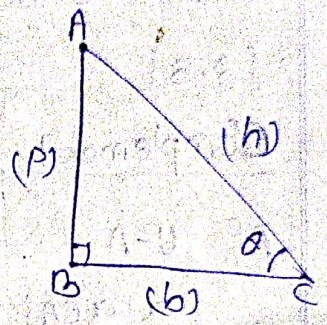
Trigonmetric function:

(i) Sin
(ii) Cos
(iii) Tan
(iv) Cot
(v) Sec
(vi) Cosec

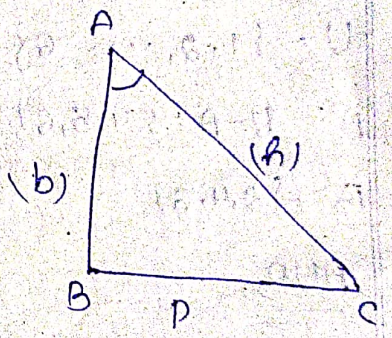
$\sin\theta = \frac{p}{h}$ $\quad$ $\cot\theta = \frac{b}{p}$

$\cos\theta = \frac{b}{h}$ $\quad$ $\sec\theta = \frac{h}{b}$

$\tan\theta = \frac{p}{b}$ $\quad$ $\operatorname{cosec}\theta = \frac{h}{p}$



By Pythagoras theorm



$h^2 = p^2 + b^2$

$\Rightarrow h = \sqrt{p^2 + b^2}$

$h^2 = h^2 \sin^2\theta + h^2 \cos^2\theta$

$\Rightarrow \cancel{h^2} = \cancel{h^2}(\sin^2\theta + \cos^2\theta)$

$\Rightarrow 1 = \sin^2\theta + \cos^2\theta$

$\sec^2\theta - \tan^2\theta = 1$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\sec^2\theta - \tan^2\theta$

$= \left(\frac{h}{b}\right)^2 - \left(\frac{p}{b}\right)^2 = \frac{h^2}{b^2} - \frac{p^2}{b^2} = \frac{h^2 - p^2}{b^2} = \frac{\cancel{b^2}}{\cancel{b^2}} = 1$

$\operatorname{Cosec}^2\theta - \cot^2\theta$

$= \left(\frac{h}{p}\right)^2 - \left(\frac{b}{p}\right)^2$

$= \frac{h^2}{p^2} - \frac{b^2}{p^2} = \frac{h^2-b^2}{p^2} = \frac{\cancel{p^2}}{\cancel{p^2}} = 1$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\theta = 1$

$\Rightarrow \frac{1}{2} + \cos^2\theta = 1$

$\Rightarrow \frac{1}{2} - 1 = \cos^2\theta \Rightarrow \frac{2-1}{2} = \cos^2\theta \Rightarrow \frac{1}{2} = \cos^2\theta \Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \frac{\cancel{1}}{\cancel{\sqrt{2}}} \times \frac{\cancel{\sqrt{2}}}{\cancel{1}} = 1$

$\sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \boxed{\sec\theta = \sqrt{1+\tan^2\theta}}$

$\tan^2\theta = \sec^2\theta - 1$

$\Rightarrow \boxed{\tan\theta = \sqrt{\sec^2\theta - 1}}$

$\operatorname{Cosec}^2\theta = 1 + \cot^2\theta$

$\Rightarrow \operatorname{Cosec}\theta = \sqrt{1+\cot^2\theta}$

$\cot^2\theta = \operatorname{Cosec}^2\theta - 1$

$\Rightarrow \cot\theta = \sqrt{\operatorname{Cosec}^2\theta - 1}$

$\sin\theta = \frac{1}{2}$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{1}{2}\right)^2} = \sqrt{1-\frac{1}{4}} = \sqrt{\frac{4-1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\sec\theta = \cancel{\sin\theta}\ \sqrt{1+\tan^2\theta} = \sqrt{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \sqrt{1+\frac{1}{3}} = \sqrt{\frac{3+1}{3}}$

$= \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$

$\operatorname{Cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{1}{2}} = 2$

$\cot\theta = \sqrt{\operatorname{cosec}^2\theta - 1}$

$= \sqrt{(2)^2 - 1} = \sqrt{4-1} = \sqrt{3}$

$\pi = 3.14$ $\quad \pi = 180^\circ$ $\quad \frac{\pi}{6} = 30^\circ$

$= \frac{22}{7}$ $\quad \frac{\pi}{2} = 90^\circ$ $\quad 2\pi = 360^\circ$

$= \sqrt{10}$ $\quad \frac{\pi}{3} = 60^\circ$ $\quad \frac{\pi}{4} = 45^\circ$

θ	0	30	45°	60°	90°
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
$\operatorname{cosec}\theta$	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1

$\sin(90^\circ - \theta) = \cos\theta$

$\cos(90^\circ - \theta) = \sin\theta$

$\tan(90^\circ - \theta) = \cot\theta$

$\cot(90^\circ - \theta) = \tan\theta$

$\sec(90^\circ - \theta) = \operatorname{cosec}\theta$

$\operatorname{cosec}(90^\circ - \theta) = \sec\theta$

$\sin(90^\circ + \theta) = \cos\theta$

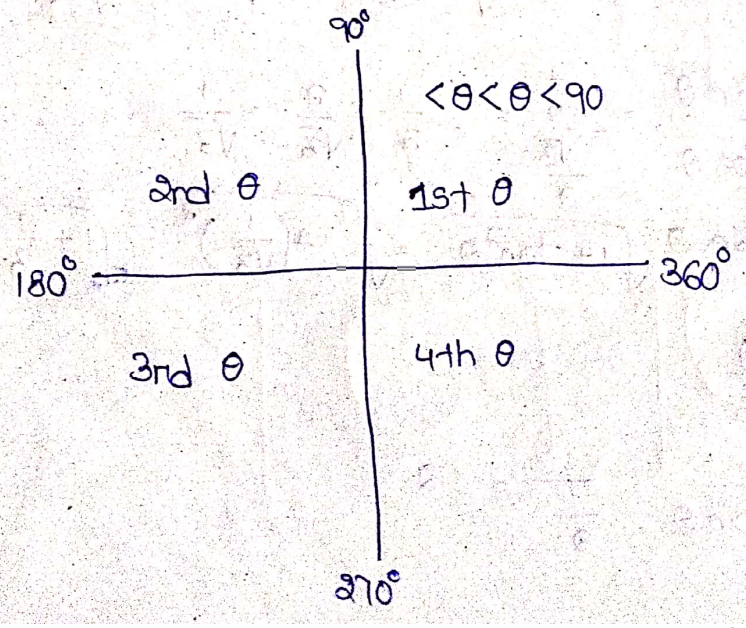
$\cos(90^\circ + \theta) = -\sin\theta$

$\tan(90^\circ + \theta) = -\cot\theta$

$\cot(90^\circ + \theta) = -\tan\theta$

$\sec(90^\circ + \theta) = -\operatorname{cosec}\theta$

$\operatorname{cosec}(90^\circ + \theta) = \sec\theta$



$\sin(180^\circ - \theta) = \sin\theta$

$\cos(180^\circ - \theta) = -\cos\theta$

$\tan(180^\circ - \theta) = -\tan\theta$

$\cot(180^\circ - \theta) = -\cot\theta$

$\sec(180^\circ - \theta) = -\sec\theta$

$\operatorname{cosec}(180^\circ - \theta) = \operatorname{cosec}\theta$

$\sin(270^\circ - \theta) = -\cos\theta$

$\cos(270^\circ - \theta) = -\sin\theta$

$\tan(270^\circ - \theta) = \cot\theta$

$\cot(270^\circ - \theta) = \tan\theta$

$\sec(270^\circ - \theta) = -\operatorname{cosec}\theta$

$\operatorname{cosec}(270^\circ - \theta) = -\sec\theta$

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

$\sec(-\theta) = \sec\theta$

$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$\sin(180^\circ + \theta) = -\sin\theta$

$\cos(180^\circ + \theta) = -\cot\theta$

$\tan(180^\circ + \theta) = -\tan\theta$

$\cot(180^\circ + \theta) = \cot\theta$

$\sec(180^\circ + \theta) = -\sec\theta$

$\operatorname{cosec}(180^\circ + \theta) = -\operatorname{cosec}\theta$

$\sin(270^\circ + \theta) = -\cos\theta$

$\cos(270^\circ + \theta) = \sin\theta$

$\tan(270^\circ + \theta) = -\cot\theta$

$\cot(270^\circ + \theta) = -\tan\theta$

$\sec(270^\circ + \theta) = \operatorname{cosec}\theta$

$\operatorname{cosec}(270^\circ + \theta) = -\sec\theta$

$< \quad >$

$a < b$ if $a - b < 0$ or $(-ve)$

$a > b$ if $a - b > 0$ or $(+ve)$

$\sin(1792) = \sin(360^\circ \times 5 - 8)$

$= \sin(-8)$

$= -\sin 8$

$\sin(-1134) = \sin(360 \times 3 + 54)$

$= -\sin 54^\circ$

$\sin(-3888) = \sin(360^\circ \times 10 + 288)$

$= -\sin 288^\circ$

$= \sin(360^\circ - 72^\circ) = \sin 72^\circ = \sin(90^\circ - 18^\circ)$

$= \cos 18^\circ$

$\tan(7265) = \tan(360 \times 20 + 65)$

$= \tan 65^\circ$

$= \tan(90^\circ - 25^\circ)$

$= \cot 25^\circ$

$\cos(5397^\circ) = \cos(360 \times 14 + 357)$

$= \cos 357$

$= \cos(360^\circ - 3)$

$= \cos 3$

$\sec(1938) = \sec(360 \times 5 + 138)$

$= \sec 138$

$= \sec(90^\circ + 48) = -\operatorname{cosec} 48^\circ$

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$

$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$\cot(A-B) = \dfrac{\cot A \cdot \cot B + 1}{\cot B - \cot A}$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$\cot(A+B) = \dfrac{\cot A \cdot \cot B - 1}{\cot B + \cot A}$

$\sin 75^\circ = \sin(45^\circ + 30^\circ)$

$= \sin 45^\circ \cdot \cos 30^\circ + \cos 45^\circ \cdot \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\sin 15^\circ = \sin(45^\circ - 30^\circ)$

$= \sin 45^\circ \cdot \cos 30^\circ - \cos 45^\circ \cdot \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\sin 2A = \sin(A+A)$

$= \sin A \cdot \cos A + \cos A \cdot \sin A$

$= 2 \sin A \cdot \cos A = \frac{2 \tan A}{1 + \tan^2 A}$

$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\cos^2 A$

$= \frac{1 - \tan^2 A}{1 + \tan^2 A}$

$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

$(\sin A)^2 = \sin^2 A$

By putting $\sin^2 A = 1 - \cos^2 A$

Putting $\cos^2 A = 1 - \sin^2 A$

$\cos 2A = 2\cos^2 A - 1$

$\Rightarrow 1 + \cos 2A = 2\cos^2 A$

$\Rightarrow \frac{1+\cos 2A}{2} = \frac{\cancel{2}\cos^2 A}{\cancel{2}}$

$\Rightarrow \frac{1+\cos 2A}{2} = \cos^2 A$

$\cos 2A = 1 - 2\sin^2 A$

$\Rightarrow 1 - \cos 2A = 2\sin^2 A$

$\Rightarrow \frac{1-\cos 2A}{2} = \frac{\cancel{2}\sin^2 A}{\cancel{2}}$

$\Rightarrow \frac{1-\cos 2A}{2} = \sin^2 A$

$$\tan^2 A = \frac{1-\cos 2A}{1+\cos 2A}$$

$$\sin A = \sqrt{\frac{1-\cos 2A}{2}}$$

$$\cos A = \sqrt{\frac{1+\cos 2A}{2}}$$

$$\tan A = \sqrt{\frac{1-\cos 2A}{1+\cos 2A}}$$

Put $\frac{\theta}{2} = A$

$$\sin A = \sin\frac{\theta}{2} = \sqrt{\frac{1-\cos 2A}{2}}$$

$$= \sin\left(\frac{\theta}{2}\right)^2 = \frac{1-\cos 2A}{2}$$

$$= \frac{\sin^2\theta}{4} = \frac{1-\cos 2A}{2}$$

$$= 2\sin^2\theta = 4 - 4\cos 2A$$

$$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$(\cos A + \sin A)^2 = \cos^2 A + \sin^2 A + 2 \cdot \cos A \cdot \sin A$$

$$= 1 + \sin 2A$$

$$(\cos A - \sin A)^2 = \cos^2 A + \sin^2 A - 2 \cos A \cdot \sin A$$

$$= \cancel{2\sin}\ 1 - \sin 2A$$

Put $A = \frac{\theta}{2}$

$$\pm\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right) = \sqrt{1+\sin\theta}$$

$$\pm\left(\cos\frac{\theta}{2} + -\sin\frac{\theta}{2}\right) = \sqrt{1-\sin\theta}$$

$$\sqrt{1+\sin\theta} = \pm\cos\frac{\theta}{2} + \sin\frac{\theta}{2}$$

$$\sqrt{1-\sin\theta} = \pm\cos\frac{\theta}{2} - \sin\frac{\theta}{2}$$

$$\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)^2 = \cos^2\theta + \sin^2\theta \pm 2\cos\frac{\theta}{2} \cdot \sin\frac{\theta}{2}$$

$$\boxed{\cos\frac{\theta}{2} + \sin\frac{\theta}{2} = 1 + \sin\theta}$$

$$\cos 22\tfrac{1}{2} + \sin 22\tfrac{1}{2}$$

$$= \sqrt{1 + \sin 2 \cdot 22\tfrac{1}{2}}$$

$$\cos\frac{45}{2} + \sin\frac{45}{2}$$

$$= \sqrt{1 + \sin 45}$$

$$= \sqrt{1 + \frac{1}{\sqrt{2}}} = \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}}} = \sqrt{1+1} = \sqrt{2}$$

Prove

$$\frac{1-\cos A}{\sin A} = \tan\frac{A}{2}$$

$$\text{LHS} = \frac{1-\cos^2 A/2}{\sin 2A \cdot \frac{A}{2}} = \frac{2\sin^2\frac{A}{2}}{2\sin A/2 \cdot \cos A/2}$$

$$= \tan\frac{A}{2}$$

$\tan\left(\frac{\pi}{4}+\theta\right)$

$$= \frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\pi/4\cdot\tan\theta}$$

$$= \frac{\frac{\tan\pi+4\tan\theta}{4}}{\frac{1-\tan^2\pi\theta}{4}}$$

$$= \frac{\frac{\tan\pi+4\tan\theta}{4}}{\frac{4-\tan^2\pi\theta}{4}}$$

$$= \frac{\tan\pi+4\tan\theta}{\cancel{4}} \times \frac{\cancel{4}}{4-\tan^2\pi\theta}$$

$$= \frac{\tan\pi+4\tan\theta}{4-\tan^2\pi\theta}$$

$\tan\left(\frac{\pi}{4}-\theta\right)$

$$= \frac{\tan\pi/4-\tan\theta}{1+\tan\pi/4\times\tan\theta} = \frac{1-\tan\theta}{1+\tan\theta}$$

<u>Imp</u>

$$\tan\left(\frac{\pi}{4}+\theta\right) = \frac{1+\tan\theta}{1-\tan\theta} = \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$$

$$\tan\left(\frac{\pi}{4}-\theta\right) = \frac{1-\tan\theta}{1+\tan\theta} = \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$$

$\sin(A+B) + \sin(A-B) = 2 \cdot \sin A \cdot \cos B$

$\sin(A+B) - \sin(A-B) = 2 \cdot \cos A \cdot \sin B$

$\cos(A+B) + \cos(A-B) = 2 \cdot \cos A \cdot \cos B$

$\cos(A+B) - \cos(A-B) = 2 \cdot \sin A \cdot \sin B$

$\sin(A+B) + \sin(A-B) = 2 \cdot \sin A \cdot \cos B$

$A+B = C$ —— (I)

$A-B = D$ —— (II)

$2A = C+D$

$A = \frac{C+D}{2}$

$A+B = C$ —— (I)

$A-B = D$ —— (II)

$(-) \quad (+) \quad (-)$

$2B = C-D$

$B = \frac{C-D}{2}$

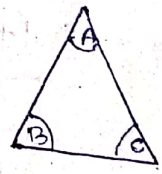
$\sin C + \sin D = 2 \cdot \sin \frac{C+D}{2} \cdot \cos \frac{C-D}{2}$

$\sin C - \sin D = 2 \cdot \cos \frac{C+D}{2} \cdot \sin \frac{C-D}{2}$

$\cos C + \cos D = 2 \cdot \cos \frac{C+D}{2} \cdot \cos \frac{C-D}{2}$

$\cos C - \cos D = -2 \cdot \sin \frac{C+D}{2} \cdot \sin \frac{C-D}{2}$

$A+B+C = 180^\circ$ or π



$\Rightarrow A+B = \pi - C, \; 180^\circ - C$

$\Rightarrow \sin(A+B) = \sin(180^\circ - C) = \sin C$

$\cos(A+B) = \cos(180^\circ - C) = -\cos C$

Prove that in ΔABC

$\sin 2A + \sin 2B + \sin 2C = 4 \sin A \cdot \sin B \cdot \sin C$

$= 2 \sin \frac{2A+2B}{2} \cdot \cos \frac{2A-2B}{2} + 2 \sin C \cdot \cos C$

$= 2 \sin(A+B) \cdot \cos(A-B) + 2 \sin C \cdot \cos C$

$= 2 \sin C \cos(A-B) - 2 \sin C \cos(A+B)$

$= 2 \sin C [\cos(A-B) - \cos(A+B)]$

$= 2 \sin C \left[-2 \sin \frac{A-\cancel{B}+A+\cancel{B}}{2} \cdot \sin \frac{\cancel{A}-B-\cancel{A}-B}{2}\right]$

$= 2 \sin C (-2) \times \sin A \times (-\sin B)$

$= 4 \cdot \sin A \cdot \sin B$

$\cot \frac{\pi}{8} - \tan \frac{\pi}{8} = 2$

$= \frac{1}{\tan \frac{\pi}{8}} - \tan \frac{\pi}{8}$

$= \frac{1 - \tan^2 \frac{\pi}{8}}{\tan \frac{\pi}{8}} = \frac{\frac{2 \times 1}{2 \tan \frac{\pi}{8}}}{\frac{1 - \tan^2 \frac{\pi}{8}}{}} = \frac{2}{\tan \frac{\pi}{4}} = 2$ (Proved)

$\tan 54^\circ = \tan(45^\circ + 9^\circ)$

$= \frac{\tan 45^\circ + \tan 9}{1 - \tan 45^\circ \cdot \tan 9^\circ}$

$= \frac{1 + \tan 9^\circ}{1 - 1 \cdot \tan 9^\circ}$

$= \frac{1 + \tan 9^\circ}{1 - \tan 9^\circ}$

$= \frac{1 + \frac{\sin 9^\circ}{\cos 9^\circ}}{1 - \frac{\sin 9^\circ}{\cos 9^\circ}} = \frac{\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ}}{\frac{\cos 9^\circ - \sin 9^\circ}{\cos 9^\circ}}$

$= \frac{\cos 9^\circ + \sin 9}{\cancel{\cos 9^\circ}} \times \frac{\cancel{\cos 9^\circ}}{\cos 9^\circ - \sin 9^\circ}$

$= \frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ}$

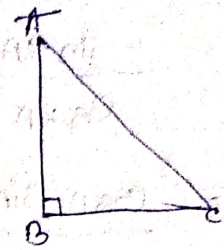
$\tan 10 + \tan 35 + \tan 10 \cdot \tan 35 = 1$

$\tan 45^\circ = \tan(35+10)$

$\Rightarrow 1 = \dfrac{\tan 35^\circ + \tan 10^\circ}{1 - \tan 35^\circ \cdot \tan 10^\circ}$

$\Rightarrow 1 - 1 \cdot \tan 35^\circ \cdot \tan 10^\circ = \tan 35^\circ + \tan 10^\circ$

$\Rightarrow \tan 10^\circ + \tan 35^\circ + \tan 10 \cdot \tan 35 = 1$ (Proved)



$A+B+C = 180^\circ$

$A+B = 180^\circ - C$

$\cos(A+B) = \cos(180^\circ - C)$

$= \cos(A+B) = -\cos C$ —— (i)

$A+B = 180^\circ - C$

$\sin(A+B) = \sin(180^\circ - C)$

$\sin(A+B) = \sin C$ —— (ii)

$\cos(A+B) + \sin C = \sin(A+B) - \cos C$ (Proved)

$A+B+C = 180^\circ$

$\Rightarrow B+C = 180^\circ - A$

$\Rightarrow \dfrac{B+C}{2} = \dfrac{180^\circ - A}{2} = 90^\circ - \dfrac{A}{2}$

$\Rightarrow \tan \dfrac{B+C}{2} = \cot \dfrac{A}{2}$ (Proved)

$A+B+C = 180^\circ$

$\Rightarrow A+B = 180^\circ - C$

$\Rightarrow \tan(A+B) = \tan(180^\circ - C)$

$\Rightarrow \tan A + \tan B = -\tan C$

$\Rightarrow \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = -\tan C$

$\Rightarrow \tan A + \tan B = -\tan C - \{-\tan C\} \cdot \tan A \cdot \tan B$

$\Rightarrow \tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$ (Proved)

$\sec 2A - \tan 2A$

$= \frac{1}{\cos 2A} - \tan 2A$

$= \frac{1}{\cos 2A} - \frac{\sin 2A}{\cos 2A}$

$= \frac{1 - \sin 2A}{\cos 2A} = \frac{(\cos A - \sin A)^2}{\cos^2 A - \sin^2 A} = \frac{(\cos A - \sin A)^{\cancel{2}}}{(\cos A + \sin A)\cancel{(\cos A - \sin A)}}$

$= \frac{\cos A - \sin A}{\cos A + \sin A}$ (Proved)

Excercise - 4 (a)

(1) (i) $\cos 270^\circ = +ve$

(ii) $\sec 73^\circ = +ve$

(iii) $\sin 302^\circ = -ve$

(iv) $\operatorname{cosec} 159^\circ = +ve$

(v) $\sec 199^\circ = -ve$

(vi) $\operatorname{cosec} 186^\circ = +ve$

(vii) $\cos 315^\circ = +ve$

(viii) $\cot 375^\circ = +ve$

(2) $\cos \theta = a + \frac{1}{a}$ doesn't have a solution if $a \neq 0$

$\cos \theta \neq a + \frac{1}{a}$

$(\cos \theta)^2 = \left(a + \frac{1}{a}\right)^2$

$= \left(a - \frac{1}{a}\right)^2 - 4 \cdot \cancel{a} \cdot \frac{1}{\cancel{a}}$

$= \left(a - \frac{1}{a}\right)^2 + 4$

$\sin\theta + \sin\phi = a$

$\cos\theta + \cos\phi = b$

Prove that $\frac{\tan\frac{\theta+\phi}{2}}{} = \frac{a}{b}$

$$\Rightarrow \frac{a}{b} = \frac{\sin\theta \cdot \sin\phi}{\cos\theta + \cos\phi}$$

$$\Rightarrow \frac{a}{b} = \frac{2 \cdot \sin\frac{\theta+\phi}{2} \cdot \cos\frac{\theta+\phi}{2}}{2 \cdot \cos\frac{\theta+\phi}{2} \cdot \cos\frac{\theta-\phi}{2}}$$

$$\Rightarrow \frac{a}{b} = \frac{\sin\frac{\theta+\phi}{2}}{\cos}$$

$$\Rightarrow \frac{a}{b} = \tan\frac{\theta+\phi}{2}$$

$\sin A + \sin 3A + \sin 5A$

$= \sin 3A (\sin 5A + \sin A)$

$= \sin 3A \left(2 \cdot \sin\frac{5A+A}{2} \cdot \cos\frac{5A-A}{2}\right)$

$= \sin 3A \left(2 \cdot \sin\frac{\cancel{6}^{3}A}{\cancel{2}} \cdot \cos\frac{\cancel{4}^{2}A}{\cancel{2}}\right)$

$= \sin 3A (2 \cdot \sin 3A \cdot \cos 2A)$

$= \sin 3A (1 + 2\cos 2A)$

$\sin\alpha \in [-1, 1]$

$\cos\alpha \in [-1, 1]$

$\alpha \in (a, b) \rightarrow$ open interval

$\Rightarrow a < \alpha < b$

$\alpha \in (a, b) \rightarrow$ closed interval

$\Rightarrow a \leq \alpha \leq b$

Inverse Trigonometry Functions

$\sin x = y$

$\sin^{-1} y = x$

$\sin^{-1}(\sin x) = x$

$\cos^{-1}(\cos x) = x$

$\tan^{-1}(\tan x) = x$

$\operatorname{Cosec} x = \frac{1}{\sin x} = \frac{1}{y}$

$\Rightarrow x = \sin^{-1} y \Rightarrow x = \operatorname{Cosec}^{-1} \frac{1}{y}$

$\sin^{-1} y = \operatorname{Cosec}^{-1} \frac{1}{y}$

$\tan^{-1} y = \cot^{-1} \frac{1}{y}$

$\cos^{-1} y = \sec^{-1} \frac{1}{y}$

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow \sin x = \sqrt{1 - \cos^2 x}$

$\sin^{-1} x = \cos^{-1} \sqrt{1 - x^2}$

$\sin^{-1} x = \tan^{-1} \frac{x}{\sqrt{1 - x^2}}$

$\sin \frac{\pi}{6} = \frac{1}{2}$

$\Rightarrow \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$

Let $y = \sin^{-1} x \Rightarrow x = \sin y$

$= \cos y = \sqrt{1 - \sin^2 y}$

$= \sqrt{1 - x^2}$

$y = \cos^{-1} = \sqrt{1 - x^2}$

$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$

$\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$

$2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$

$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\frac{x+y+z-xyz}{1-xy-yz-zx}$

$2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2} = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2}$

$\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

$\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} + -y\sqrt{1-x^2}\right)$

$\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$

$\cos^{-1}x - \cos^{-1}y = \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right.$

$2\sin^{-1}x = \sin^{-1}\left(2\cdot x\sqrt{1-x^2}\right)$

$2\cos^{-1}x = \cos^{-1}\left(2x^2-1\right)$

$3\sin^{-1}x = \sin^{-1}\left(3x-4x^3\right)$

$3\cos^{-1}x = \cos^{-1}\left(4x^3-3x\right)$

$3\tan^{-1}x = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$